



empirical developmental methods to supplement the filter theory customarily employed. The purpose of this note is to present a technique for simplifying the empirical development.

The electrical performance of a fixed tuned coaxial band-pass filter is completely determined by the resonant frequency of the cavities, the unloaded  $Q$ 's of the cavities, the input/output couplings to specified impedance levels, and the interstage couplings.<sup>2</sup> For multiresonator filters having unloaded  $Q$ 's an order of magnitude greater than the minimum unloaded  $Q$ 's required to realize the desired filter response shape, the bandwidth of the filter is primarily determined by the interstage couplings. To design a tunable band-pass filter with constant bandwidth, the frequency sensitivities of the interstage couplings must be controlled.

The electrical performance of an interstage coupling mechanism can be described quantitatively by a coefficient of coupling. This coefficient of coupling can be conveniently measured by known techniques.<sup>3</sup> It can be shown that

$$K \cong \frac{\Delta f}{f_0} \quad \text{at any frequency } f_0,$$

where

$$K = \text{coefficient of coupling,} \\ \Delta f = \text{coupling bandwidth.}$$

If the absolute bandwidth (not per cent bandwidth) of the tunable filter must be kept constant at all frequencies, then  $K$  must vary inversely with frequency and  $\Delta f$  must be independent of frequency.

A convenient mechanism for achieving interstage coupling at microwave frequencies is an aperture. Apertures are often preferable to probes, loops, or direct connections since no soldering is required and apertures can be readily machined to precision tolerances. The most convenient aperture is circular since it is easiest to fabricate. It can be shown that for small lossless circular apertures in thin walls

$$\Delta f \cong C f^2 \left[ 2 \cos^2 \left( \frac{2\pi f L}{v} \right) - \sin^2 \left( \frac{2\pi f L}{v} \right) \right],$$

where

$$\Delta f = \text{coupling bandwidth} \\ f = \text{frequency and } C \text{ is a constant,} \\ L = \text{distance from centerline of aperture} \\ \text{to plane of short-circuit,} \\ v = \text{velocity of propagation in free space.}$$

Taking  $d(\Delta f)/df$  and equating to zero,

$$\cos^2 \left( \frac{2\pi f_m L}{v} \right) - \left( \frac{2\pi f_m L}{v} \right) \cos \left( \frac{2\pi f_m L}{v} \right) \sin \left( \frac{2\pi f_m L}{v} \right) = \frac{1}{3},$$

where  $f_m$  = frequency of maximum coupling. Letting

$$\theta = \frac{2\pi f_m L}{v}, \quad \cos^2 \theta - \theta \cos \theta \sin \theta = \frac{1}{3}.$$

<sup>2</sup> M. Dishal, "Dissipative band-pass filters," *Proc. IRE*, vol. 37, pp. 1050-1069; September, 1949.  
<sup>3</sup> M. Dishal, "Alignment and adjustment of synchronously tuned multiple-resonator-circuit filters," *Proc. IRE*, vol. 39, pp. 1448-1455; November, 1951.

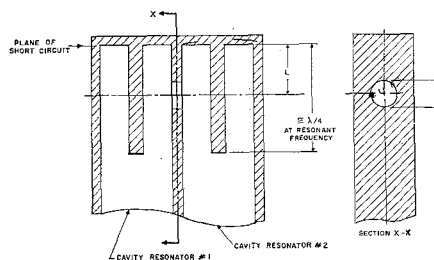


Fig. 1—Coaxial band-pass filter interstage coupling.

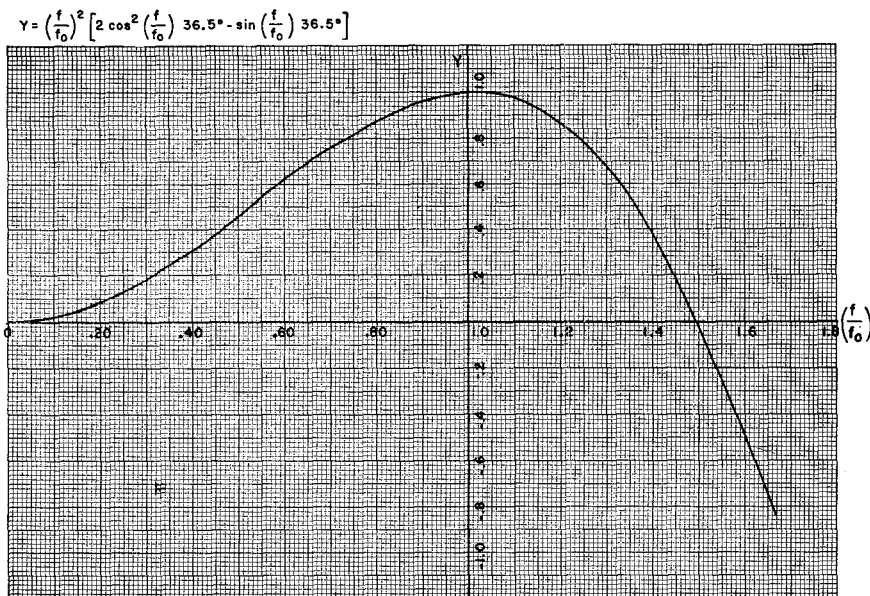


Fig. 2—Frequency sensitivity of circular aperture coupling bandwidth.

This is a transcendental equation which must be solved graphically. It can be shown that for maximum coupling bandwidth  $\theta \cong 36^\circ$ .

To design a tunable coaxial band-pass filter using circular apertures as interstage coupling mechanisms, it is necessary to specify both the aperture location ( $L$ ) and the aperture diameters ( $d$ ) to determine the coupling bandwidth. (See Fig. 1.) Using the theory previously developed,  $L$  is chosen to make  $\theta = 36$  electrical degrees at the center frequency of the tuning range. This reduces the design problem to that of determining the aperture diameter.  $d$  is best determined empirically and it is a relatively simple matter to obtain a plot of  $\Delta f$  vs  $d$  by measuring the coefficients of coupling at the center of the tuning range. Aperture diameters are then chosen to realize the coupling bandwidths required for the particular filter response shape that is desired.

The technique of locating circular interstage coupling apertures in tunable coaxial band-pass filters for maximum coupling bandwidths (i.e., minimum frequency sensitivity of coupling bandwidth) has been successfully implemented for various filters. These filters have employed  $\lambda/4$  resonators with cylindrical center conductors coaxial to either cylindrical, square, or rectangular outer conductors. The technique has worked for filters using both contacting fingers and noncontacting shorts.

$$\text{A plot of } Y = \left( \frac{f}{f_0} \right)^2 \left[ 2 \cos^2 \left( \frac{f}{f_0} \right) 36.5^\circ - \sin^2 \left( \frac{f}{f_0} \right) 36.5^\circ \right]$$

is shown in Fig. 2. In practice, the aperture centerline is usually located between 36.5 and 40.5 electrical degrees from the plane of the short circuit. It can be seen that  $Y$  is not a symmetrical function of  $(f/f_0)$  and that the high end of the tuning range becomes more frequency sensitive when tuning over wide ranges. The simple theory presented also neglects open-end fringing capacitance and the fact that contacting fingers make the resonators compound transmission lines. The theory is most valid when the coupling bandwidth varies as the cube of the aperture diameters. It is less applicable for very small apertures and/or thick walls which cause appreciable attenuation through the apertures. It also breaks down when the aperture diameter approaches the size of the cavity outer conductor.

The theory presented can also be extended to noncircular apertures, loop couplings, and/or design for constant percentage bandwidth filters. It is highly recommended as a means of substantially reducing the design/development cycle required for tunable coaxial band-pass filters.

RICHARD M. KURZROK  
Surface Communications Systems Lab.  
Radio Corporation of America  
New York, N. Y.